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Quaternion-based color difference measure for removing impulse noise in color images

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***Abstract*—This paper introduces a new color difference measure based on quaternion representation and a filtering algorithm for impulse noise removal in color images. Simulations and comparisons demonstrate that our measure and algorithm have excellent performance in detecting and removing impulse noise.**

***Keywords*—Color image denoising, Quaternion , Color difference, noise detector**

I. Introduction

Digital images are easily corrupted with noise caused by hardware or scenario during acquisition, transmission or other image processing stage. It has a huge effect to subsequent image processing operations for reducing corrupted pixels. Thus, image denoising has been proven to play a vital role in many image processing tasks such as edge detection, image enhancement and segmentation, as well as pattern recognition [1].

Recently, quaternion representation of color images became a hot research topic because it can process three-dimensional (3D) color channels in a whole. The quaternion has been demonstrated to be a robust tool for distinguishing the impulse noise because the quaternion results of normal and noise pixels are significantly different. It has good performance for removing impulse noise in color images [2].

Many algorithms have been proposed for removing impulse noise. The well-known order-statistic vector filter, vector median filter (VMF) and its extensions [3]–[5], have been proven to have good effects in removing impulse noise. However, VMF may blur fine details and lines while filtering images. Jin and Li [6] proposed a quaternion switching vector median filter (QSVMF) to efficiently detect the color impulse noise and proved its simplicity and complexity. Later, Jin et al. used chromaticity for the quaternion based color image denoising [7]. In [8], the order-statistic information was used for detecting impulse noise while searching edge directions. But it does not work for the case that the color noise is in or close to gray lines. Geng et al. proposed a quaternion switching filter (QSF) [9] to detect the impulse noise using chromaticity and intensity. However, QSF has difficulty to measure the changes of chromaticity and intensity in color images.

To over the above mentioned problems, this paper introduces a new color difference measure to correctly distinguish the difference between two color pixels. Based on this new measure, we propose a new quaternion switching filtering algorithm for removing impulse noise in color images. Simulations and comparisons are provided.

This paper is organized as follows: Section II briefly reviews the quaternion and its properties as background. Section III proposes the new color difference measure. Section IV introduces a filtering algorithm based on this method. Section V presents simulations and comparisons. Section VI concludes this paper.

II. Preliminary

A. Quaternion

A quaternion proposed by Hamilton is a four-dimensional (4D) complex format defined by [10]

$$q = a + bi + cj + dk \tag{1}$$

where a, b, c and d are coefficients, i, j and k are complex operators obeying the following rules:

$$\begin{cases} i^2 = j^2 = k^2 = ijk = -1 \\ ij = k, \ jk = i, \ ki = j \\ ji = -k, \ kj = -i, \ ik = -j \end{cases}$$

Compared with two-dimensional (2D) complex numbers, a quaternion has one real component and four imaginary components. It can be considered as an extension of two dimensional complex numbers. When the real coefficient $a = 0$, q becomes a *pure quaternion*. The modulus and conjugate of a quaternion are defined as follows,

$$|q| = \sqrt{a^2 + b^2 + c^2 + d^2} \tag{2}$$

$$\overline{q} = a - bi - ci - dk \tag{3}$$

A quaternion with a unit modulus is called a unit quaternion. In addition to the hypercomplex form, a quaternion also has other representations such as the polar form [11].

$$\begin{aligned} q &= |q|^{e^{\mu\theta}} \\ &= |q|(cos\theta + \mu sin\theta) \end{aligned} \tag{4}$$

where μ and θ are the eigenaxis and eigenangle of q, and μ is also called a unit pure quaternion.

B. Quaternion representation of color images

A color image in the RGB space has three different color channels: the red, green and blue channels. Because a pure quaternion also has three imaginary numbers, a color image can be represented by a pure quaternion as follows.

$$q_{x,y} = r_{x,y}i + g_{x,y}j + b_{x,y}k \tag{5}$$

where x, y denote the position of a pixel where $r, g, b \in [0, 255]$ represent the pixel intensity values in the red, green, blue channels, respectively. Thus, a color pixel that has three components in the RGB space is converted into one quaternion element to be processed.

C. Quaternion rotation

In a 3D space, RXR^* represents a rotation of vector X to the axis ξ with an angle of 2θ [12], [13]. The unit pure quaternion R means that each component has the same value. For example, $R = G = B$ in the RGB space is called a "gray line". We can see that both the RGB space and pure quaternion R are 3D vectors. Thus, for any vector X, RXR^* denotes a 3D rotation of X by $180°$ to the gray line $R|_{\mu,\theta}$. If X represents a color pixel, X and its rotation form RXR^* have opposite directions but the same distance from the pure quaternion R. For example, q_2 shown in Fig. 1. Therefore, the new vector $X + RXR^*$, a summation of the color pixel vector X and its rotation RXR^*, should lie in the gray line.

III. New quaternion color difference measure

Removing impulse noise from images includes two stages, noise detection and image filtering. Noise detection uses a difference measure to distinguish the normal and noisy pixels. This section will introduce a new color difference measure in the quaternion domain to improve the detection accuracy of impulse noise.

A. Problem and analysis

The quaternion color difference measure (QCDM) is the similarity metric to measure the deference of two color pixels in the quaternion domain. Assume two color pixels are represented in the quaternion form, $q_1 = r_1 i + g_1 j + b_1 k$ and $q_2 = r_2 i + g_2 j + b_2 k$, the quaternion rotation can be used to measure their difference. Utilizing the quaternion properties, a new vector is defined as $q_3 = q_1 + Rq_2R^* = r_3 i + g_3 j + b_3 k$. When q_1 and q_2 are equal or close to each other, q_3 should be in or near to the gray line. Otherwise, q_3 should be far away from the gray line. Based on this, a QCDM was designed to estimate the difference between q_1 and q_2 [6].

$$Q(q_3) = Q(q_1 + Rq_2R^*) =$$
$$(r_3 - \tfrac{r_3+g_3+b_3}{3})i + (g_3 - \tfrac{r_3+g_3+b_3}{3})j + (b_3 - \tfrac{r_3+g_3+b_3}{3})k \tag{6}$$

Obviously, module $|Q(q_1 + Rq_2R^*)|$ denotes the color difference between pixels q_1 and q_2. It is effective to detect impulse noise such as Salt and Pepper noise and random value noise. For example, three color pixels $q_1 = 20i + 80j + 190k$, $q_2 = 30i + 70j + 200k$ and $q_3 = 170i + 180j + 90k$, then $Q(q_1, q_2) = 16.33$, $Q(q_2, q_3) = 193.05$ and $Q(q_1, q_3) = 187.08$. When q_1 and q_2 are similar, the result approaches zero. If q_3 is a noise pixel, the color differences $Q(q_1, q_3)$ and $Q(q_2, q_3)$ are large. These results show the effectiveness of this QCDM. If q_1 and q_2 are different, $C = q_1 + Rq_2R^*$ is far away from the gray line. From the mathematical viewpoint, the module of vector C is quite large and equivalent to $|Q(q_1 + Rq_2R^*)|$.

However, the existing measure in Eqn. (6) is unable to distinguish different pixels in the gray line that represents an axis in the RGB space. The pixels in the gray line are monochrome with values $R = G = B$. For instance, for a black pixel $q_4 = 0i + 0j + 0k$ and a white pixel $q_5 = 255i + 255j + 255k$, QCDM considers both pixels are the same because $|Q(q_4 + Rq_5R^*)| = 0$ but this is not true. Next, we propose a new QCDM to address this problem

B. New quaternion color difference measure

Fig.1 shows that two pixels q_1 and q_2 are in the gray line but their modules are different. In the quaternion space, their modules can be regarded as intensity.

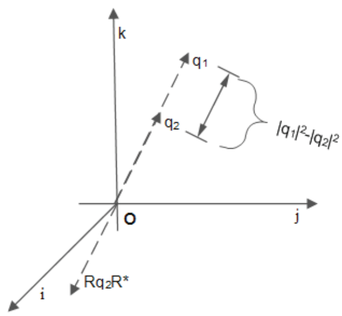


Fig. 1: The proposed quaternion color difference measure.

Borrowing the idea from the HSI space, the intensity of a color pixel can be represented as $Int = \frac{1}{3}(R + G + B)$. We then introduce a new quaternion color difference measure defined by

$$d_(q1, q2) = cd_(q1, q2) + id(q1, q2) \tag{7}$$

where

$$cd_(q1, q2) = Q(q_1 + Rq_2R^*)$$
$$id_(q1, q2) = |Int(q_1)|^2 - |Int(q_2)|^2$$

The proposed measure is the summation of the chromaticity and intensity differences of two color pixels. When the pixels are in the gray line, we are able to distinguish them because their intensities are taken into account in the proposed measure. Next, we use this measure to propose a new algorithm for removing impulse noise.

IV. New quaternion switching filtering algorithm

In this section, we propose a quaternion switching filtering algorithm to remove impulse noise in color images using the quaternion color difference measure proposed in Section III. The proposed algorithm uses the quaternion color difference measure for detecting noise pixels and switching filtering for noise removal. If a pixel is detected as a noise pixel, it will be replaced by the filtering result; otherwise it keeps unchanged. Fig. 2 shows the flowchart of the proposed quaternion switching filtering algorithm.

In Fig. 2, the algorithm first represents a noised color image in a quaternion form that maps the color image from the RGB space into the quaternion domain. It uses a noise detector using the proposed quaternion color difference measure to detect impulse noise in the color image. By calculating the quaternion

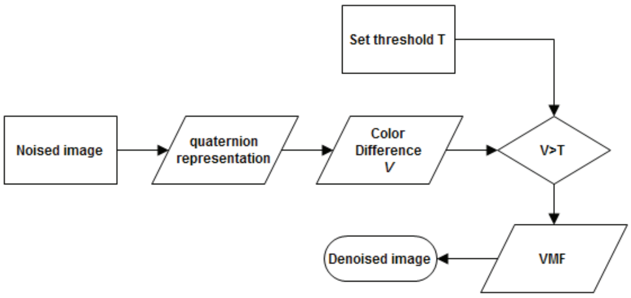


Fig. 2: Flowchart of the new quaternion switching filtering algorithm.

difference between a pixel with its surrounding pixels, we obtain a result V. Comparing V with a predefined threshold T can identify whether the pixel is a noisy pixel or not. If yes, the algorithm will filter it using a VMF.

Suppose that a 3×3 filtering window is represented by

$$\begin{bmatrix} q1 & q2 & q3 \\ q4 & q5 & q6 \\ q7 & q8 & q9 \end{bmatrix}$$

To calculate the quaternion color differences between the center pixel and its neighboring pixels, we defined eight differences in different directions within the filtering window, namely $V_i, i \in \{1, ..., 8\}$.

$$V_1 = \frac{1}{2}(d(q_4, q_5) + (q_5, q_6))$$
$$V_2 = \frac{1}{2}(d(q_3, q_5) + (q_5, q_7))$$
$$V_3 = \frac{1}{2}(d(q_2, q_5) + (q_5, q_8))$$
$$V_4 = \frac{1}{2}(d(q_1, q_5) + (q_5, q_9))$$
$$V_5 = \frac{1}{2}(d(q_4, q_5) + (q_5, q_2))$$
$$V_6 = \frac{1}{2}(d(q_2, q_5) + (q_5, q_6))$$
$$V_7 = \frac{1}{2}(d(q_6, q_5) + (q_5, q_8))$$
$$V_8 = \frac{1}{2}(d(q_4, q_5) + (q_5, q_8))$$

If the center pixel q_5 is a normal pixel, at least one of V_i will be extremely small. When the center pixel q_5 is a corrupted pixel, all differences V_i in eight directions are quite large. Therefore, it is necessary to set a threshold T to determine whether the center pixel is a noise pixel or not. If the minimal value of V_i is greater than T, the pixel is considered as noisy and will be replaced by VMF; Otherwise, the pixel keeps unchanged.

Obviously, VMF uses quaternion-based distance measure as defined by,

$$q_{x,y}^{VMF} = \arg \min_{q_t \in \{q_1, q_2, \dots, q_N\}} \sum_{s=1}^{N} ||q_s - q_t|| \qquad (8)$$

where x and y denote the coordinates in the image, s and t are pixel indexes within a filtering window.

The filtered result can be written as follows:

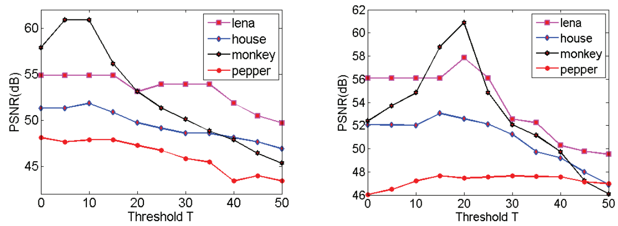
$$\widetilde{q}_{x,y} = \begin{cases} q_{x,y}^{VMF} & V > T \\ q_{x,y} & Otherwise \end{cases} \qquad (9)$$

V. Simulation Results

After discussing the parameter selection, this section provides several experimental and comparison results to show the performance of the proposed quaternion swathing filtering algorithm. The test images come from the USC-SIPI image database[1]. The denoising results are quantitatively evaluated by PSNR.

A. Parameter optimization

Because the threshold T plays a significant role of the denoising performance of the proposed filtering algorithm. Here, we fist show how to select the optimal threshold T.



(a) (b)

Fig. 3: Optimization of threshold T. The denoising results of different original images with (a) 5% and (b) 15% Salt & Peper noise, respectively.

We test images with different contents and noise levels. The denoising results shown in Fig. 3 change dramatically. When $T = 20$, we can obtain a good denoising results while balancing the content images and different noise levels. Therefore, we set $T = 20$ for the rest experiments in this paper.

TABLE I: PSNR results of the images before and after denoising using the proposed filtering algorithm with different types and levels of impulse noise

Noise type	SP		RV		RRV	
Noise Level	before	after	before	after	before	after
5%	24.2850	**47.4493**	48.0251	**52.5530**	50.0818	**54.9874**
10%	24.3942	**45.0341**	45.6981	**48.8953**	48.7667	**52.1834**
15%	24.5053	**43.8691**	44.3180	**47.8926**	47.5222	**50.6799**
20%	24.6258	**42.6493**	42.6955	**46.4661**	46.6073	**49.3274**
25%	24.7382	**41.8268**	41.9094	**45.4337**	45.7564	**48.3202**
30%	24.8675	**41.1232**	41.1344	**44.6594**	45.0369	**47.6912**
35%	24.9950	**40.8367**	40.5286	**43.8953**	44.3745	**47.0523**
40%	25.1130	**40.3129**	40.2216	**43.4719**	43.6710	**46.5310**

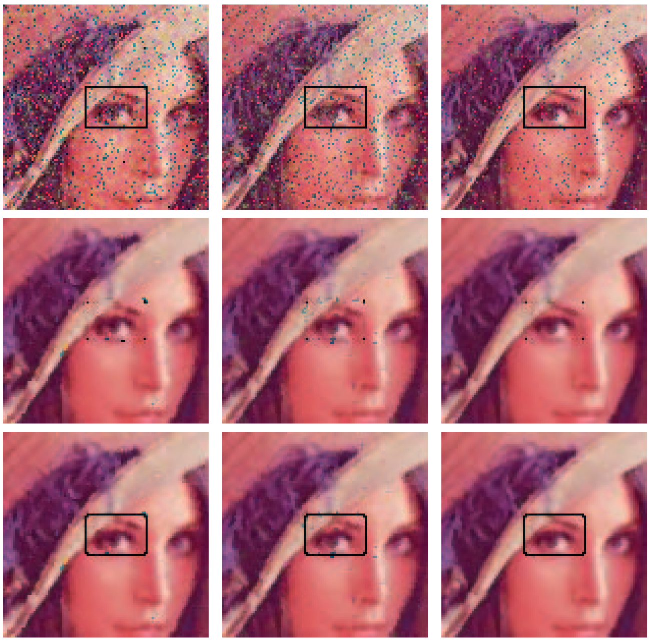
Table I shows the denoising performance of the proposed filtering algorithm. We use three types of noise: Salt & Pepper (SP) noise, random value (RV) impulse noise, and the related random value (RRV) impulse noise. The noise levels ranges from 5% to 40%. As the noise level increases, the performance of the proposed algorithm changes dynamically.

B. Noise images with a fine rectangle

An image may contain special symbols such as rectangle shown in Fig. 4. These symbols are regular fine details and parts of image contents. A good denoising algorithm should remove noise while keeping these symbols. Fig. 4 compares

[1]The USC-SIPI image database is located in http://sipi.usc.edu/database.

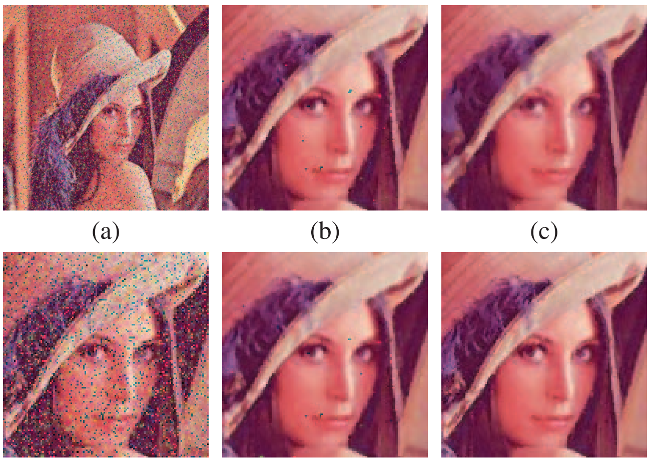
the proposed algorithm with the classical vector median filter for removing different types of impulse noise in an image with a rectangle. As can been seen, the proposed algorithm well preserves the rectangle while removing noise. It outperforms the classical vector median filter.



(a) (b) (c)

Fig. 4: Comparison of image denoising when the image contains a fine rectangle. The top row shows noise images with 15%; the middle row shows the denoising results of the Vector Median Filter; and the bottom row shows the results of the proposed algorithm. (a) SP; (b) RV; and (c) RRV.

C. Removing Impulse noise



(a) (b) (c)

(d) (e) (f)

Fig. 5: Comparison of removing the salt & pepper noise. (a) noise image; (b) QSVMF; (c) QSF; (d) zoomed noise image; (e) VMF; (f) The proposed algorithm.

Fig. 5 compares different algorithms for removing the salt & pepper noise. Table II compares those algorithms under different types and levels of noise. We can observe that the denoising performance of these algorithm becomes worse with the noise level increasing. For all cases, the proposed algorithm obtains the best denoising results and outperforms other methods.

TABLE II: Comparison of different denoising algorithms

Noise type	Method	5%	10%	15%	20%
SP	VMF	55.0586	53.7895	50.4835	47.4503
	QSVMF	56.1378	54.0569	50.2620	47.9219
	QSF	56.0672	54.0129	53.1812	52.6717
	Proposed	**57.0014**	**55.9293**	**54.8530**	**54.3532**
RV	VMF	58.4283	57.4178	57.3180	56.5344
	QSVMF	58.7407	58.3682	58.0805	57.8106
	QSF	59.0775	58.4281	58.1365	57.7586
	Proposed	**63.0014**	**62.5298**	**61.3194**	**60.6667**
RRV	VMF	59.3721	58.7617	57.6242	56.8271
	QSVMF	58.8723	58.7407	58.3091	58.3091
	QSF	59.5978	59.2199	58.9396	58.6763
	Proposed	**62.8361**	**62.2302**	**60.2405**	**59.3841**

VI. Conclusion

In this paper, we proposed a new color difference measure to detect the pixel differences in quaternion domain. Using this measure, we further introduced a switching filtering algorithm for removing impulse noise in color images, including the Salt & Pepper noise, random value noise and related random value noise. Experiments and comparisons have demonstrated that the proposed measure and algorithm have excellent performance in detecting and removing impulse noise.

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